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Neutrino survival probabilities in magnetic fields

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Abstract

We show that, for Majorana neutrinos propagating in a constant magnetic field, the flavour survival probabilities for left-handed neutrinos is the same as for right-handed neutrinos, i.e., $P^M(\nu_{\alpha L} \to \nu_{\alpha L}) = P^M(\nu_{\alpha R} \to \nu_{\alpha R})$, where $\alpha = e, \mu, \tau$, whereas in the Dirac case the corresponding probabilities $P^D(\nu_{\alpha L} \to \nu_{\alpha L})$ and $P^D(\bar{\nu}_{\alpha R} \to \bar{\nu}_{\alpha R})$ are in general different. This might lead to a novel way to search for the nature of neutrinos. We also discuss how this relation for Majorana neutrinos gets modified when the magnetic field is not constant. However, if matter effects become important the relation does not hold anymore.

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I. INTRODUCTION

In the past few years neutrino physics has generated tremendous interest and has been looked upon as a window to physics beyond the standard model. Recent results from Super-Kamiokande [1] have boosted this search for new physics. Both the solar and atmospheric neutrino anomalies agree well with the neutrino oscillation hypothesis [2] (for fits to the solar neutrino data see Ref. [3]), which requires neutrinos to be massive. For recent reviews see, e.g., Ref. [4]. However, it should be kept in mind that neutrino flavour oscillations is not the only possibility to describe the data. It may be that massive neutrinos have non-zero magnetic moments (MM) and electric dipole moments (EDM), which could play an important role in the solar neutrino puzzle if large magnetic fields are present in the interior of the sun [5]. In this framework, the attractive scenario of resonant spin – flavour transitions [6] has received attention and good fits to the solar neutrino data have been obtained (for recent fits see Refs. [7] and for reviews see Ref. [8]). In solutions of the solar neutrino puzzle with non-zero MMs and EDMs, it is assumed that the solar neutrinos interact with the magnetic field in the sun to produce right-handed neutrinos due to a helicity flip. Right-handed neutrinos are sterile in the case of Dirac neutrinos and behave like antineutrinos in the Majorana case. In the latter case, a flavour transition simultaneous with the helicity flip produces the suppression in solar neutrino detection with elastic neutrino – electron scattering.

So far, it is not known if neutrinos possess MMs and/or EDMs. The most stringent laboratory bounds come from elastic neutrino – electron scattering: for $\bar{\nu}_e$ reactor neutrinos the limit is $1.8 \times 10^{-10} \mu_B$ [9] whereas for $\overset{(-)}{\nu}_\mu$ the limit is $7.4 \times 10^{-10} \mu_B$ [10], where μ_B is the Bohr magneton. More stringent limits are obtained with astrophysical considerations [11]. For a collection of references on limits on neutrino MMs see also Ref. [12].

In this paper we focus on neutrino survival probabilities. Apart from the three active neutrino flavours (ν_α with $\alpha = e, \mu, \tau$), we allow an arbitrary number of additional neutrinos of the sterile type (ν_s) [13]. In vacuum, the survival probabilities of left-handed and right-handed (anti)neutrinos are equal:

$$P^D(\nu_{\alpha L} \to \nu_{\alpha L}) = P^D(\bar{\nu}_{\alpha R} \to \bar{\nu}_{\alpha R}) \quad \text{and} \quad P^M(\nu_{\alpha L} \to \nu_{\alpha L}) = P^M(\nu_{\alpha R} \to \nu_{\alpha R}) \,, \tag{1}$$

where the superscripts D and M refer to Dirac and Majorana neutrinos, respectively. These equalities are not valid if matter effects [14] become important, but matter effects in neutrino oscillations do not distinguish between the Dirac and Majorana nature [15]. Here we concentrate on the situation that matter effects are negligible but the MM and EDM interaction of neutrinos with magnetic fields becomes important. Thus our discussion does not apply to solar neutrinos. We will show that for constant magnetic fields the equality (1) remains valid for Majorana neutrinos whereas in general it gets lost for Dirac neutrinos. This is to be contrasted with the matter effects. We will also consider a situation where the magnetic field depends on $\vec{x}$ where this equality for Majorana neutrinos persists. Though our discussion will be rather formal we hope that it can shed some light on a possible distinction between Dirac and Majorana neutrinos. Up to now such efforts are mainly concentrated on neutrinoless double beta decay [16].

Our paper is organized as follows. In Section II, we discuss the general formalism for an oscillating neutrino (both, left and right-handed neutrinos) propagating in an external

electromagnetic field, in the absence of matter effects, and prove the equality (1) for Majorana neutrinos in a constant magnetic field. In Section III we study some conditions where equality (1) also holds for Dirac neutrinos, but we show that it is violated in the general Dirac case. In Section IV we consider non-constant magnetic fields and in Section V we summarize our results.

II. GENERAL FORMALISM

In the following we will work with n neutrino flavours or types. We will take into account neutrino mixing and electromagnetic interactions through MMs and EDMs but we will not consider matter effects.

A. DIRAC CASE

Let us assume that the neutrino MMs, EDMs and transition MMs and EDMs are given for the neutrino mass eigenfields ν_j. Then the MM and EDM interaction of the Dirac neutrinos is expressed by the Hamiltonian density

$$\mathcal{H}^D_{\rm em} = \frac{1}{2}\bar{\nu}(\mu + id\gamma_5)\sigma_{\alpha\beta}F^{\alpha\beta}\nu \quad \text{with} \quad \mu^\dagger = \mu,\ d^\dagger = d \tag{2}$$

being the magnetic moment and electric dipole moment matrices, respectively. $F^{\alpha\beta}$ is the antisymmetric electromagnetic field tensor. Whereas ν_j denotes the fields in the mass basis, we denote the chiral fields in the flavour basis where the charged lepton matrix is diagonal by ν_L, ν_R. The mass matrix M in the neutrino mass term

$$-\mathcal{L}^D_m = \bar{\nu}_R M \nu_L + \text{h.c.} \tag{3}$$

is bidiagonalized by

$$U_R^\dagger M U_L = \hat{M} \tag{4}$$

such that

$$\nu_{\alpha L} = \sum_j U_{L\,\alpha j}\,\nu_{jL} \quad \text{and} \quad \nu_{\alpha R} = \sum_j U_{R\,\alpha j}\,\nu_{jR}\,, \tag{5}$$

where the indices α denote the neutrino flavours or types. The diagonalizing matrix U_L is the usual neutrino mixing matrix.

For massive Dirac neutrinos with mixing, the Hamiltonian in the *flavour basis* describing the neutrino interacting with a magnetic field is given as [6,17,18]

$$H^D_\nu = \begin{pmatrix} \frac{1}{2E_\nu}U_L\hat{M}^2U_L^\dagger & -B_+U_L(\mu+id)U_R^\dagger \\ -B_-U_R(\mu-id)U_L^\dagger & \frac{1}{2E_\nu}U_R\hat{M}^2U_R^\dagger \end{pmatrix}, \tag{6}$$

where the upper half of the matrix H^D_ν corresponds to negative helicity while the lower half corresponds to positive helicity. Assuming that the neutrino is propagating along the z direction, the magnetic fields $B_\pm$ are defined as

$$B_\pm = B_x \pm iB_y = Be^{\pm i\beta} \quad \text{with} \quad B = \sqrt{B_x^2 + B_y^2}\,. \tag{7}$$

In the approximation we are working with, the longitudinal magnetic field is negligible. The neutrino energy is denoted by E_ν. Note that for the right-handed Dirac neutrino states there is no preferred flavour basis because these states do not couple to the charged leptons. The Hamiltonian matrix $H_{\bar\nu}^D$ for Dirac antineutrinos is obtained by making the replacements

$$U_{L,R} \to U_{L,R}^*\,, \quad \mu \to -\mu^T = -\mu^*\,, \quad d \to -d^T = -d^* \tag{8}$$

in the matrix (6). The superscript $*$ on the MM and EDM matrices indicates complex conjugation of all elements of the matrices. The upper half of $H_{\bar\nu}^D$ maps onto positive and the lower half onto negative helicity.

The Hamiltonian (6) is re-expressed in the mass basis as $\tilde H_\nu$, where

$$\tilde H_\nu = \begin{pmatrix} \frac{1}{2E_\nu}\hat M^2 & -B_+(\mu + id) \\ -B_-(\mu - id) & \frac{1}{2E_\nu}\hat M^2 \end{pmatrix}, \tag{9}$$

and the corresponding Hamiltonian for antineutrinos is given by

$$\tilde H_{\bar\nu} = \begin{pmatrix} \frac{1}{2E_\nu}\hat M^2 & B_+(\mu^* + id^*) \\ B_-(\mu^* - id^*) & \frac{1}{2E_\nu}\hat M^2 \end{pmatrix}. \tag{10}$$

In order to obtain the survival probabilities of neutrinos (which have negative helicity) and antineutrinos (which have positive helicity), one needs to diagonalize the matrices $\tilde H_\nu$ and $\tilde H_{\bar\nu}$, respectively. Observing that

$$J^\dagger \tilde H_{\bar\nu} J = \tilde H_\nu^* \quad \text{with} \quad J = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}, \tag{11}$$

where $\mathbf{1}$ is the $n \times n$ unit matrix, we find that H_ν^D and $H_{\bar\nu}^D$ have the same eigenvalues $E_1, \ldots, E_{2n}$. Furthermore, there are unitary matrices W_ν and $W_{\bar\nu}$ such that

$$W_\nu^\dagger \tilde H_\nu W_\nu = W_{\bar\nu}^\dagger \tilde H_{\bar\nu} W_{\bar\nu} = \mathrm{diag}(E_1, E_2, ..., E_{2n})\,, \tag{12}$$

which, according to Eq.(11), are related by

$$W_{\bar\nu} = J W_\nu^*\,. \tag{13}$$

The survival probabilities for Dirac neutrinos (helicity $= -1$) and Dirac antineutrinos (helicity $= +1$) are then given as

$$P^D(\nu_{\alpha L} \to \nu_{\alpha L}) = \left| \sum_{j=1}^{2n} \left| U_{\alpha j}^\nu \right|^2 e^{-iE_j L} \right|^2 \tag{14}$$

and

$$P^D(\bar\nu_{\alpha R} \to \bar\nu_{\alpha R}) = \left| \sum_{j=1}^{2n} \left| U_{\alpha j}^{\bar\nu} \right|^2 e^{-iE_j L} \right|^2, \tag{15}$$

respectively. In the above probability expressions, L is the distance between neutrino source and detector. From the $2n \times 2n$ matrices U^{ν} and $U^{\bar{\nu}}$ diagonalizing H_{ν}^{D} and $H_{\bar{\nu}}^{D}$, respectively, we only need the first n lines labelled by the neutrino flavours or types, which can be expressed by U_L and $W \equiv W_{\nu}$:

$$\begin{array}{rl} 1 \leq j \leq n: & U_{\alpha j}^{\nu} = \sum_{k=1}^{n} U_{L\,\alpha k} W_{kj}\,, \quad U_{\alpha j}^{\bar{\nu}} = \sum_{k=1}^{n} U_{L\,\alpha k}^{*} W_{n+k\,j}^{*}\,, \\ n+1 \leq j \leq 2n: & U_{\alpha j}^{\nu} = \sum_{k=1}^{n} U_{L\,\alpha k} W_{k\,n+j}\,, \; U_{\alpha j}^{\bar{\nu}} = \sum_{k=1}^{n} U_{L\,\alpha k}^{*} W_{n+k\,n+j}^{*}\,. \end{array} \tag{16}$$

B. MAJORANA CASE

For Majorana neutrinos we start with the same electromagnetic Hamiltonian density (2) as in the Dirac case, except that the factor 1/2 is replaced by 1/4 to account for the charge conjugation property $(\nu_j)^c = \nu_j$ of the Majorana fields. Then the Hamiltonian H_M, corresponding to H_{ν}^{D}, is given by

$$H_M = \begin{pmatrix} \frac{1}{2E_\nu} U_L \hat{M}^2 U_L^{\dagger} & -B_+ U_L(\mu + id) U_L^T \\ -B_- U_L^{*}(\mu - id) U_L^{\dagger} & \frac{1}{2E_\nu} U_L^{*} \hat{M}^2 U_L^T \end{pmatrix}. \tag{17}$$

Note that U_R in H_{ν}^{D} (6) is replaced by U_L^{*} in the Majorana case. Furthermore, we have to keep in mind that the matrices μ and d are antisymmetric, i.e.,

$$\mu^T = \mu^* = -\mu\,, \quad d^T = d^* = -d \tag{18}$$

in H_M. This equation formulates the well known properties of Majorana neutrinos that only transition moments are non-zero and that μ and d are purely imaginary. The latter property allows to check immediately that

$$J^{\dagger} H_M J = H_M^{*} \tag{19}$$

holds (compare with Eq.(11)).

Denoting the diagonalizing $2n \times 2n$ unitary matrix of H_M by U_M, the survival probabilities of Majorana neutrinos with negative and positive helicities are given by

$$P^M(\nu_{\alpha L} \to \nu_{\alpha L}) = \left| \sum_{j=1}^{2n} |U_{M\,-\alpha j}|^2 \, e^{-iE_j L} \right|^2 \tag{20}$$

and

$$P^M(\nu_{\alpha R} \to \nu_{\alpha R}) = \left| \sum_{j=1}^{2n} |U_{M\,+\alpha j}|^2 \, e^{-iE_j L} \right|^2 , \tag{21}$$

respectively. The subscript $-$ of U_M in Eq.(20) indicates that the index α refers to the first n lines of this matrix (negative helicity), whereas in Eq.(21) the subscript $+$ indicates the use of the last n lines (positive helicity).

The relation (19) allows to be more specific with respect to the matrix U_M.

Lemma: The matrix U_M which diagonalizes H_M can be chosen to be of the form

$$U_M = \begin{pmatrix} A & B^* \\ B & -A^* \end{pmatrix} .$$

Furthermore, diagonalizing H_M with this U_M we obtain

$$E_j = E_{n+j} \quad \text{for} \quad j = 1, \ldots, n.$$

Proof of the lemma: Suppose we have an eigenvector ψ of H_M, i.e.,

$$H_M \psi = E \psi \,. \tag{22}$$

Then with Eq.(19) we find that

$$H_M J \psi^* = J \left(J^\dagger H_M J \right) \psi^* = J H_M^* \psi^* = E J \psi^* \quad \text{and} \quad J\psi^* \perp \psi \,. \tag{23}$$

This observation allows to construct an orthonormal basis of eigenvectors of H_M in the following way. Starting with an eigenvector ψ_1 normalized to unit length and with eigenvalue E_1, then $J\psi_1^*$ is orthogonal to ψ_1 and has the same eigenvalue. Next we can find a normalized eigenvector ψ_2 with eigenvalue E_2 such that $\psi_2 \perp \{\psi_1, J\psi_1^*\}$. Then it is easy to show that $J\psi_2^*$ is orthogonal to all three previously constructed eigenvectors. We continue by finding ψ_3 with eigenvalue E_3 orthogonal to all four previously constructed eigenvectors and so on. After n steps, two orthonormal systems $\{\psi_j\}_{j=1,\ldots,n}$ and $\{J\psi_j^*\}_{j=1,\ldots,n}$ with the same eigenvalues $E_1, \ldots, E_n$ are found which together form an orthonormal basis of eigenvectors of H_M. Thus U_M is given by

$$U_M = \left(\psi_1 \cdots \psi_n \; J\psi_1^* \cdots J\psi_n^* \right) , \tag{24}$$

which is of the form announced in the lemma. $\Box$

An immediate consequence of the lemma and of Eqs.(20) and (21) is the following theorem.

Theorem 1: Without matter effects and in a constant magnetic field the survival probabilities for left-handed and right-handed Majorana neutrinos are equal, i.e.,

$$P^M(\nu_{\alpha L} \to \nu_{\alpha L}) = P^M(\nu_{\alpha R} \to \nu_{\alpha R}) = \left| \sum_{j=1}^{n} \left(|A_{\alpha j}|^2 + |B_{\alpha j}|^2 \right) e^{-iE_j L} \right|^2 .$$

III. ASYMMETRIES OF SURVIVAL PROBABILITIES

Let us define an asymmetry

$$\Delta_\alpha^D = P^D(\nu_{\alpha L} \to \nu_{\alpha L}) - P^D(\bar\nu_{\alpha R} \to \bar\nu_{\alpha R}) \tag{25}$$

for every Dirac neutrino flavour or type α. Note that we have just discussed in Theorem 1 that the corresponding asymmetry for Majorana neutrinos,

$$\Delta_\alpha^M = P^M(\nu_{\alpha L} \to \nu_{\alpha L}) - P^M(\nu_{\alpha R} \to \nu_{\alpha R}) \,, \tag{26}$$

vanishes with the assumptions in this theorem. If we set $\mu = d = 0$ and assume vanishing matter effects, then we have vacuum oscillations and both asymmetries (25) and (26) are zero. Clearly, the asymmetry (25) is not a CP asymmetry due to a KM type of phase [19]. If it is non-zero it is because the presence of a magnetic field represents a CP-violating situation.

As a passing remark, matter effects would induce such an asymmetry for the same reason: background matter is not CP-invariant or, in other words, neutrinos and antineutrinos interact differently with matter. The matter potentials enter in the diagonal of the Hamiltonian matrices as follows:

$$\begin{array}{rl} (V_L, 0) & \text{for Dirac neutrinos,} \\ (-V_L, 0) & \text{for Dirac antineutrinos,} \\ (V_L, -V_L) & \text{for Majorana neutrinos,} \end{array} \tag{27}$$

with the diagonal matrix $V_L = \sqrt{2} G_F \, \mathrm{diag}(\mathrm{N}_\alpha)$, where, for ordinary matter, $N_e = n_e - n_n/2$, $N_{\mu,\tau} = -n_n/2$, $N_s = 0$ and n_e and n_n are the electron and neutron densities, respectively.

However, there is a fundamental difference between matter effects and the effects of MMs, EDMs and a magnetic field: For $B_\pm = 0$, but matter effects becoming important, one has $\Delta_\alpha^D = \Delta_\alpha^M \neq 0$ in general, whereas with $V_L = 0$ but $B_\pm \neq 0$ and constant magnetic field one has $\Delta_\alpha^D \neq 0$ in general, as we will see, but $\Delta_\alpha^M = 0$.

In view of Theorem 1 and in order to elaborate under which conditions Majorana and Dirac neutrinos behave differently, it is necessary to study the asymmetry (25) in more detail. The following theorem describes under which sufficient conditions the asymmetry is still zero.

Theorem 2: For the case of Dirac neutrinos, without matter effects and with a constant magnetic field, if the electric and magnetic moment matrices obey the proportionality $\mu = cd$ (or $d = c\mu$), where c is a real number, then the survival probabilities of a neutrino with flavour (type) α is equal to the survival probability of the corresponding antineutrino, i.e., the asymmetry (25) is zero.

Corollary: If $d = 0$ or $\mu = 0$ or $n = 1$ the asymmetry (25) is zero.

Proof of the theorem: We define (see Eq.(7))

$$e^{i\beta}(1 + ic) \equiv z = |z| e^{i\zeta} \,. \tag{28}$$

We can write $\tilde{H}_\nu$ (9) as

$$\tilde{H}_\nu = \begin{pmatrix} \frac{1}{2E_\nu} \hat{M}^2 & -Bz\mu \\ -Bz^*\mu & \frac{1}{2E_\nu} \hat{M}^2 \end{pmatrix} . \tag{29}$$

The phase ζ can be removed by the unitary transformation

$$H'_\nu \equiv \begin{pmatrix} \mathbf{1} & 0 \\ 0 & \mathbf{1}e^{i\zeta} \end{pmatrix} \tilde{H}_\nu \begin{pmatrix} \mathbf{1} & 0 \\ 0 & \mathbf{1}e^{-i\zeta} \end{pmatrix} = \begin{pmatrix} \frac{1}{2E_\nu}\hat{M}^2 & -B|z|\mu \\ -B|z|\mu & \frac{1}{2E_\nu}\hat{M}^2 \end{pmatrix} = \begin{pmatrix} H_1 & -H_2 \\ -H_2 & H_1 \end{pmatrix} . \tag{30}$$

The individual block matrices H_1 and H_2 are independent Hermitian matrices.

We consider the eigenvector equations

$$\begin{aligned} (H_1 - H_2)X_j &= E_j X_j \, , \\ (H_1 + H_2)Y_j &= E_{n+j} Y_j \, , \end{aligned} \tag{31}$$

where $j = 1, \ldots, n$. The sets $\{X_j\}_{j=1,\ldots,n}$ and $\{Y_j\}_{j=1,\ldots,n}$ form orthonormal bases of eigenvectors of the Hermitian matrices $H_1 - H_2$ and $H_1 + H_2$, respectively. Therefore, the structure of W_ν is given by

$$W_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} X_1 & \cdots & X_n & Y_1 & \cdots & Y_n \\ e^{-i\zeta}X_1 & \cdots & e^{-i\zeta}X_n & -e^{-i\zeta}Y_1 & \cdots & -e^{-i\zeta}Y_n \end{pmatrix} \equiv \begin{pmatrix} C & D \\ e^{-i\zeta}C & -e^{-i\zeta}D \end{pmatrix} . \tag{32}$$

Furthermore, using the relations (16) and the expressions (14) and (15) for the Dirac neutrino survival probabilities, Theorem 2 follows. □

Now we want to show that Theorem 2 describes an exceptional situation and that indeed in general the asymmetry Δ_α^D is different from zero. It is sufficient to see this in the case of vanishing neutrino masses in H_ν^D (6) and $H_{\bar{\nu}}^D$ (8). Defining a matrix [18]

$$\lambda = \mu - id \, , \tag{33}$$

we notice that this is a completely general matrix which can be bidiagonalized with unitary matrices R and S:

$$\lambda = R\hat{\lambda}S^\dagger \quad \text{with} \quad S = (x_1, \ldots, x_n) \, , \quad R = (y_1, \ldots, y_n) \, , \tag{34}$$

where $\hat{\lambda}$ is diagonal and positive and $\{x_j\}_{j=1,\ldots,n}$ and $\{y_j\}_{j=1,\ldots,n}$ are orthonormal bases. Dropping the neutrino masses we get the Hamiltonian matrix

$$H_\nu^D = -B \begin{pmatrix} 0 & e^{i\beta}S\hat{\lambda}R^\dagger \\ e^{-i\beta}R\hat{\lambda}S^\dagger & 0 \end{pmatrix} , \tag{35}$$

which has the following eigenvectors:

$$\begin{aligned} \phi_j &= \frac{1}{\sqrt{2}} \begin{pmatrix} x_j \\ e^{-i\beta}y_j \end{pmatrix} \quad \text{with eigenvalue} \quad -B\hat{\lambda}_j \, , \\ \psi_j &= \frac{1}{\sqrt{2}} \begin{pmatrix} x_j \\ -e^{-i\beta}y_j \end{pmatrix} \quad \text{with eigenvalue} \quad B\hat{\lambda}_j \, . \end{aligned} \tag{36}$$

These eigenvectors of the Hamiltonian matrix (35) form the matrix W_ν (12), and thus with Eqs.(14), (15) and (16) we obtain

$$\begin{aligned} P^D(\nu_{\alpha L} \to \nu_{\alpha L}) &= \left| \sum_{j=1}^n |x_{\alpha j}|^2 \cos(B\hat{\lambda}_j L) \right|^2 , \\ P^D(\bar{\nu}_{\alpha R} \to \bar{\nu}_{\alpha R}) &= \left| \sum_{j=1}^n |y_{\alpha j}|^2 \cos(B\hat{\lambda}_j L) \right|^2 . \end{aligned} \tag{37}$$

From these expressions it is obvious that in general the probabilities $P^D(\nu_{\alpha L} \to \nu_{\alpha L})$ and $P^D(\bar{\nu}_{\alpha R} \to \bar{\nu}_{\alpha R})$ are different, because the orthonormal bases $\{x_j\}_{j=1,\ldots,n}$ and $\{y_j\}_{j=1,\ldots,n}$ are independent of each other.

To elaborate this in more detail we consider the case of two neutrino flavours ($n = 2$). Since we neglect here the neutrino masses all phases in λ (33) can be removed and the matrices S and R (34) are characterized by the angles θ and θ', respectively:

$$S = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = (x_1, x_2)\,, \quad R = \begin{pmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{pmatrix} = (y_1, y_2)\,. \tag{38}$$

Inserting x_j and y_j into the survival probabilities (37), it is evident that $P^D(\nu_{\alpha L} \to \nu_{\alpha L}) \neq P^D(\bar{\nu}_{\alpha R} \to \bar{\nu}_{\alpha R})$ holds as long as $\hat{\lambda}_1 \neq \hat{\lambda}_2$ and $\cos^2\theta \neq \cos^2\theta'$.

IV. MAJORANA NEUTRINOS AND z-DEPENDENT MAGNETIC FIELDS

Even if we assume that $V_L = 0$ but B_x, B_y depend on z, the survival probabilities for left-handed and right-handed Majorana neutrinos will in general be different, i.e., Theorem 1 will not hold anymore. This provides some obstacle for an application of the results of this paper with the aim to find a way to distinguish between the Dirac and Majorana nature of neutrinos. We briefly discuss the general formalism for the case of z-dependent H_M. One needs to find $2n$ linearly independent solutions of the differential equation

$$i\frac{d\varphi(z)}{dz} = H_M(z)\varphi(z)\,. \tag{39}$$

The survival probabilities depend now also on the locations z_0 of the neutrino source and z_1 of the neutrino detection. Assuming to know a complete orthonormal set $\{\varphi_j(z)\}_{j=1,\ldots,2n}$ of solutions of Eq.(39), we can formulate the transition and survival probabilities as

$$\begin{aligned} P^M(\nu_{\alpha L}(z_0) \to \nu_{\beta L}(z_1)) &= \left| \sum_{j=1}^{2n} \varphi^*_{-\alpha j}(z_0)\varphi_{-\beta j}(z_1) \right|^2, \\ P^M(\nu_{\alpha R}(z_0) \to \nu_{\beta R}(z_1)) &= \left| \sum_{j=1}^{2n} \varphi^*_{+\alpha j}(z_0)\varphi_{+\beta j}(z_1) \right|^2, \end{aligned} \tag{40}$$

and similarly those with LR and RL transitions. We use the indices $-\alpha$ and $+\alpha$ to indicate the n upper and n lower components of φ, respectively.

If the matter potential can be neglected ($V_L = 0$), the relation (19) for the Majorana Hamiltonian matrix holds, irrespective if H_M depends on z or not. It is then easy to check that, given a complete orthonormal system of solutions $\{\varphi_j(z)\}_{j=1,\ldots,2n}$, then $\{J\varphi_j^*(-z)\}_{j=1,\ldots,2n}$ is a complete orthonormal set of solutions of the differential equation (39) with $H_M(z)$ replaced by $H_M(-z)$. This suggests that with a Hamiltonian matrix H_M symmetric in z one can arrive at a situation where Theorem 1 is still valid. Indeed, using Eq.(40) and replacing $\varphi_j(z)$ by $J\varphi_j^*(-z)$, one arrives at the following relations for the Majorana transition and survival probabilities.

Theorem 3:

$$V_L = 0,\ H_M(z) = H_M(-z) \quad \Rightarrow \quad P^M(\nu_{\alpha L}(z_0) \to \nu_{\beta L}(z_1)) = P^M(\nu_{\beta R}(-z_1) \to \nu_{\alpha R}(-z_0))\,, \tag{41}$$

and, therefore, with $z_1 = -z_0$, we have

$$P^M(\nu_{\alpha L}(-z_1) \to \nu_{\alpha L}(z_1)) = P^M(\nu_{\alpha R}(-z_1) \to \nu_{\alpha R}(z_1))\,. \tag{42}$$

Therefore, Theorem 1 holds also for a z-dependent magnetic field, provided it is symmetric between the neutrino source and detection points.

Note that from Eq.(40) by exchanging complex conjugation within the absolute values, for any V_L and z-dependent magnetic field, we obtain the relations

$$\begin{aligned} P^M(\nu_{\alpha L}(z_0) \to \nu_{\beta L}(z_1)) &= P^M(\nu_{\beta L}(z_1) \to \nu_{\alpha L}(z_0))\,, \\ P^M(\nu_{\alpha R}(z_0) \to \nu_{\beta R}(z_1)) &= P^M(\nu_{\beta R}(z_1) \to \nu_{\alpha R}(z_0))\,, \end{aligned} \tag{43}$$

which express CPT invariance. However, they do not allow to relate the survival probabilities for different helicities and are, therefore, not useful in our context.

V. SUMMARY

In this paper we have studied the survival probabilities of neutrinos and antineutrinos which possess magnetic moments and electric dipole moments and propagate in magnetic fields. In particular, given a neutrino flavour (type) α, we have studied the difference $\Delta_\alpha^{D,M}$ (25,26) between the survival probabilities for $\nu_{\alpha L}$ and $\overset{(-)}{\nu}_{\alpha R}$. The bar on $\nu_{\alpha R}$ refers to Dirac neutrinos, without bar a Majorana neutrino is understood. It is well known that matter effects lead to non-zero asymmetries $\Delta_\alpha^{D,M}$, but without electromagnetic neutrino interactions one has $\Delta_\alpha^D = \Delta_\alpha^M$ expressing the fact that with neutrino oscillations one cannot distinguish between Dirac and Majorana neutrinos.

Here we have considered the opposite situation: we have assumed that matter effects are negligible but neutrinos have MMs and EDMs and propagate in magnetic fields. Assuming a *constant* magnetic field, we have shown that in this case we have $\Delta_\alpha^M = 0$ (Theorem 1), but $\Delta_\alpha^D \neq 0$ in general. However, if the MM and EDM matrices are proportional to each other in the case of Dirac neutrinos, then the asymmetry of survival probabilities is zero as well, i.e., $\Delta_\alpha^D = 0$ (Theorem 2). What happens if the magnetic field is not constant along the neutrino path between source and detector? In general, this will result in $\Delta_\alpha^M \neq 0$ for Majorana neutrinos, but if the transverse magnetic field is symmetric with respect to the center z_c of the line connecting source and detector ($B_{x,y}(z_c + z) = B_{x,y}(z_c - z)$) one still has $\Delta_\alpha^M = 0$ (Theorem 3).

The observations summarized here indicate a fundamental difference in the behaviour of Dirac and Majorana neutrinos with respect to magnetic fields, which is a consequence of the fact that the Hermitian MM and EDM matrices μ and d have to be antisymmetric for Majorana neutrinos. Further research is necessary to see if the results of this paper lead to realistic possibilities to distinguish between the Dirac and Majorana nature of neutrinos.

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